Improved Carrier Tracking of Space Telemetry Signals Using Cascaded Phase Locked Loops *

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Technical Subject Areas of ICC '95:
(1) Synchronization
(2) Satellite and Space Communications

October 6, 1994

Abstract

This paper describes a Carrier Aiding technique which employs cascaded phase-locked loops. This scheme can be useful when two unequal antennas are required to coherently demodulate the received signal, but due to an extremely weak signal-to-ratio. (SN R) level and high Doppler rates, the smaller antenna is unable to lock on the carrier on its own. In order to determine the performance for a bank of $(L-1)$ antennas when each one of them is aided by a single larger antenna, the joint pdf of the two cascaded phase error processes for these $(L-1)$ pairs must be determined. These joint pdfs as well as the carrier loop SNRs of the aided antennas are derived based on Fokker-Planck techniques. The tracking and acquisition results are verified by simulation for the case $L = 2$.

*The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

1 Introduction

Different antenna arraying techniques can be used to enhance the received signal-to-noise ratio (SNR), and hence to increase the data rate. Ideally, arraying L identical antennas increases the signal power by a factor of L^2, while the noise power increases by only a factor of L. Hence, an increase in SNR by a factor of L is potentially achievable [1,2]. Carrier arraying is one type of antenna arraying, which is used when the carrier signal is too weak to be tracked by a single phase locked loop (PLL). In this case, the carrier phase reference is estimated by using information from two or more antennas [3]. This paper is concerned with onc form of carrier arraying referred to as Carrier Aiding. In such a scheme, the carrier signal is assumed to be strong enough so that a larger antenna can synchronize to the carrier and uses its reference to down-convert tile carrier at a smaller antenna. As a result, the smaller antenna has only to coherently track the residual Doppler between the two antennas. The advantage of such a system is that it allows the smaller antenna to use a smaller bandwidth than otherwise possible, Furthermore, to enhance the performance of the decoder, the baseband signals after carrier demodulation can be combined using Baseband Combining (BBC), described in [4]. The combined baseband signal can then be demodulated by a single subcarrier and symbol loop.

In this paper, the performance of this technique is measured both in terms of SNR degradation and symbol SNR loss. Both performance measures have been discussed in details earlier [5]. Briefly, SNR degradation is defined as the ratio of the SNR at the matched filter output in the presence of nonideal synchronization to the SNR in the presence of ideal synchronization. Symbol SNR loss is defined as the symbol SNR required by a practical system to achieve a given symbol error rate (SER) divided by the symbol SNR required to achieve the same error rate in an ideal system. Here a practical system implies a system with non-ideal synchronization and an ideal system refers to a system with perfect synchronization. In the following sections, degradation and loss for the aforementioned Carrier Aiding/Baseband Combining technique are derived based on the theory of cascaded PLLs developed in [6,7,8]. The method involves first, determining the stochastic vector differential equation, known as "Itô Equation," that governs the behavior of this cascaded system.

Given that the noise is white and Gaussian, the solution to this equation is a vector Markov process which is therefore statistically completely described by the transition probability density function (pdf) and the initial distribution. Next, the Fokker-Planck (F-1') equation that describes the law of evolution of this transition pdf in time is determined. 'I'hen, an approximate analytical representation of the joint pdf for the two cascaded phase error processes, that satisfy the F-P equation, is given. For brevity, the details of the derivations are omitted and the curious reader is referred to [9], The results are then illustrated via numerical example, and are validated by computer simulations.

2 Signal & System Models

Carrier aiding using cascaded carrier loops followed by baseband combining is shown in Figure 1. As shown, the signal received at antenna i is assumed to be delayed by τ_{i1} seconds relative to antenna 1. After down-conversion to an appropriate angular IF frequency, denoted WI, the signal at antenna i is given by

$$r_i(t - \tau_{i1}) = \sqrt{2P_i}\sin[(\omega_I t - \omega_c\tau_{i1} + \delta \cdot d(t-\tau_{i1}) \cdot \mathrm{Sqr}[\omega_{sc}(t-\tau_{i1}) + \theta_{sc_i}] + \theta_{c_i}(t-\tau_{i1}))] + n_i(t) \quad (1)$$

for i= 1,2,..., L for an L-element array. P_i is the received total power in Watts (W); ω_c and θ_{c_i} are the carrier angular frequency in radians per second (rads/see) and phase in rads, respectively, and the carrier phase of the i^{th} signal is $\theta_{c_i}(t) = \theta_{c_1}(t) + \Delta\theta_i(t)$. here, $\Delta\theta_i$ accounts for the differential Doppler between signal received at antenna i and the signal received at antenna 1, the master antenna. $\mathrm{Sqr}(\omega_{sc}t + \theta_{sc})$ is the square-wave subcarrier with subcarrier angular frequency ω_{sc} in rads/sec and subcarrier phase θ_{sc} in rads. δ is the modulation index in rads. The power in the carrier is $P_{C_i} = P_i \cos^2\delta$, while the power in the modulation sidebands is $P_{D_i} = P_i - P_{c_i} = P_i \sin^2\delta$. When $\delta = 90°$, the modulation type is referred to as '(suppressed-carrier modulation. " The term "residual-carrier modulation" refers to the case when $0 \leq \delta < 90°$. The spectrum of the signal corresponding to the latter case is shown in Figure 2. The symbol stream, $d(t)$, is given by

$$d(t) = \sum_{k=-\infty}^{\infty} d_k p(t - kT) \quad (2)$$

where d_k is the ± 1 binary data for the k^{th} symbol and T is the symbol period in seconds. The baseband pulse, p(t), is unit power and limited to T seconds. The narrow-band noise $n_i(t)$ can be written as

$$n_i(t) = \sqrt{2} n_{c_i}(t) \cos(\omega_I t + \theta_{c_i}) - \sqrt{2} n_{s_i}(t) \sin(\omega_I t + \theta_{c_i}) \tag{3}$$

where $n_{c_i}(t)$ and $n_{s_i}(t)$ arc statistically independent, stationary, band-limited, white Gaussian noise processes with one-sided spectral density level N_{0i} (W/Hz) and one-sided bandwidth W_n (Hz), which is large compared to $\frac{1}{T}$.

in this scheme, antenna, 1 which is assumed to be the largest "master" antenna, locks on the signal by itself and then helps the smaller antennas, 2 through L, to track. In this case, the master antenna operates with a sufficiently large enough carrier loop bandwidth to estimate the dynamics of the signal. Using these estimates, the signal's dynamics are removed from the weaker signals received at antennas 2 through L, thus enabling those receivers to operate with smaller bandwidths than otherwise possible. The baseband signals from each antenna after carrier demodulation arc sent to a central location where they are delay compensated, weighted, combined and then processed through a chain of subcarrier loop , symbol loop and matched filter as depicted in Figure 1.

3 Carrier Aiding/BBC Performance

Assuming that the time delay for each antenna is perfectly estimated, then following the same steps of [4], the samples of the combined signal at the output of the matched filter are given by

$$v_k = \begin{cases} \sum_{i=1}^{L} \beta_i \sqrt{P_{D_i}} C_{c_i} C_{sc} d_k + n_k & d_k = d_{k-1} \\ \sum_{i=1}^{L} \beta_i \sqrt{P_{D_i}} C_{c_i} C_{sc} \left(1 - \frac{|\phi_{sy}|}{\pi}\right) d_k + n_k & d_k \neq d_{k-1} \end{cases} \tag{4}$$

where the noise n_k is a Gaussian random variable with variance given by

$$\sigma_n^2 = \frac{1}{2T} \sum_{i=1}^{L} \beta_i^2 N_{0i} \tag{5}$$

where $\beta_i = \frac{\sqrt{P_i}}{\sqrt{P_1}} \frac{N_{01}}{N_{0i}}$ are the maximal-combining ratios which maximize the SNR of the combined signal. The signal reduction functions CC, and C_{sc} are due to imperfect carrier and subcarrier synchronization and are given by [1 O]

$$C_{c_i} = \cos \phi_{c_i} \tag{6}$$

$$C_{sc} = 1 - \frac{2}{\pi}|\phi_{sc}| \tag{7}$$

where ϕ_{c_i} and ϕ_{sc} (in rads) respectively denote the carrier and subcarrier phase tracking errors. The symbol timing error, ϕ_{sy}, which affects the output only when there is a symbol transition, reduces the signal amplitude by $1 - \frac{|\phi_{sy}|}{\pi}$. Ideally, $\phi_c = \phi_{sc} = \phi_{sy} = 0$ and *(4)* reduces to the ideal matched filter output $v_k = \sum_{i=1}^{L} \sqrt{P_{D_i}} d_k + n_k$, as expected. In writing (4), it is assumed that the carrier, subcarrier, and symbol loop bandwidths are much smaller than the symbol rate so that the phase errors h,, ϕ_{sc}, and ϕ_{sy} can be modeled as constant over several symbols. Throughout this article, ϕ_{c_1} is assumed to be Tikhonov distributed, and ϕ_{sc} and ϕ_{sy} arc assumed to be Gaussian distributed. Let $p_c(\phi_{c_1})$, $p_{sc}(\phi_{sc})$, and $p_{sy}(\phi_{sy})$ denote, respectively, the carrier, subcarrier, and symbol phase error density functions. Then

$$p_c(\phi_{c_1}) = \begin{cases} \frac{\exp(\rho_{c_1}\cos\phi_{c_1})}{2\pi I_0(\rho_{c_1})}, |\phi_{c_1}| \le \pi, & \text{residual-carrier case} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $I_k(x)$ denotes the modified Bessel function of order k, and ρ_{c_1} is the "master" carrier loop SNR. Also, $p_{sc}(\phi_{sc})$ and $p_{sy}(\phi_{sy})$ are given by

$$p_i(\phi_i) = \frac{\exp(-\phi_i^2/2\sigma_i^2)}{\sqrt{2\pi}\sigma_i} \quad , \quad i = sc, sy \tag{9}$$

where σ_{sc}^2 and σ_{sy}^2 are the reciprocals of the sub carrier and symbol loop" SNRS, respectively denoted as ρ_{sc} and ρ_{sy}. The carrier, sub carrier, and symbol loop SN Rs are respectively given as [5]

$$\rho_c = \frac{P_C/N_0}{B_c}, \quad \text{residual-carrier case} \tag{10}$$

$$\rho_{sc} = \left(\frac{2}{\pi}\right)^2 \frac{P_D/N_0}{W_{sc}B_{sc}} \left(1 + \frac{1}{2E_s/N_0}\right)^{-1} \tag{11}$$

$$\rho_{sy} = \frac{P_D/N_0}{2\pi^2 W_{sy}B_{sy}} \frac{\left(\operatorname{erf}\left(\sqrt{\frac{E_s}{N_0}}\right) - \frac{W_{sy}}{2\sqrt{\pi}}\sqrt{\frac{E_s}{N_0}}\exp\left(-\frac{E_s}{N_0}\right)\right)^2}{\left(1 + \frac{E_s}{N_0}\frac{W_{sy}}{2} - \frac{W_{sy}}{2}\left[\frac{1}{\sqrt{\pi}}\exp\left(-\frac{E_s}{N_0}\right) + \sqrt{\frac{E_s}{N_0}}\operatorname{erf}\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2\right)} \tag{12}$$

where E_s/N_0 is the combined symbol SNR, $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-v^2)dv$ is the error function, and B_c, B_{sc}, and B_{sy} (in Hz) denote the single-sided carrier, sub carrier, and symbol loop bandwidths, respectively. The parameters W_{sc} and W_{sy} which denote the subcarrier and symbol window, are unitless and limited to (0, 1]. Before we can derive the performance, we need to determine the symbol SNR conditioned on ϕ_{c_i}, ϕ_{sc} and ϕ_{sy}. This conditional symbol SNR, denoted SSNR', is derived from (4), by dividing the conditional mean of the matched filter output by its conditional variance, as

$$SSNR' = \begin{cases} \dfrac{\left[\sum_{i=1}^{L} \beta_i \sqrt{E_{si}} C_{c_i} C_{sc}\right]^2}{\sum_{i=1}^{L} \beta_i^2 N_{0i}} & d_k = d_{k-1} \\[2ex] \dfrac{\left[\sum_{i=1}^{L} \beta_i \sqrt{E_{si}} C_{c_i} C_{sc} \left(1 - \frac{|\phi_{sy}|}{\pi}\right)\right]^2}{\sum_{i=1}^{L} \beta_i^2 N_{0i}} & d_k \neq d_{k-1} \end{cases} \tag{13}$$

where $E_{si} = P_{Di} T$ is the symbol energy in the i^{th} antenna, measured in Joules (J).

3.1 Symbol SNR Degradation

The SSNR degradation, defined as the ratio of SSNR in the presence of imperfect synchronization to that of the ideally combined SSNR, for this case is determined by averaging (13) over all the phase error processes and then dividing the result with the ideal combined SNR. The following expression is obtained

$$D = 10 \times \left[\log_{10} \overline{C_{sc}^2}\, \overline{C_{sy}^2} \left(\frac{\sum_{m=1}^{L} \gamma_m^2 \overline{C_{c_m}^2} + \sum_{m=1}^{L} \sum_{\substack{n=1 \\ n \neq m}}^{L} \gamma_m \gamma_n \overline{C_{c_m,c_n}}}{\left(\sum_{m=1}^{L} \gamma_m\right)^2} \right)\right] \tag{14}$$

where $\gamma_i = \frac{P_i}{P_1} \frac{N_{01}}{N_{0i}}$ are called the antenna's Gamma Factors [1], and they represent antenna i's gain-to-noise temperature ratio, denoted (G/T), normalized by (G/T) of the reference antenna, and $\overline{C_{c_i}^2}$, $\overline{C_{sc}^2}$ and $\overline{C_{sy}^2}$ are previously derived in [1]

$$\overline{C_{c_i}^2} = \frac{1}{2}\left[1 + \frac{I_2(\rho_{c_i})}{I_0(\rho_{c_i})}\right] \text{, residual-carrier case} \tag{15}$$

$$\overline{C_{sc}^2} = 1 - \sqrt{\frac{32}{\pi^3}} \frac{1}{\sqrt{\rho_{sc}}} + \frac{4}{\pi^2} \frac{1}{\rho_{sc}} \tag{16}$$

$$\overline{C_{sy}^2} = 1 - \sqrt{\frac{2}{\pi^3}} \frac{1}{\sqrt{\rho_{sy}}} + \frac{1}{4\pi^2} \frac{1}{\rho_{sy}} \tag{17}$$

$\overline{C_{c_m,c_n}}$, the joint carrier phase processes, on the other hand, has to be determined numerically from the following expression

$$\overline{C_{c_m,c_n}} = \int_{\phi_{c_n}} \int_{\phi_{c_m}} \int_{\phi_{c_1}} \cos\phi_{c_m} \cos\phi_{c_n} \left[\frac{p(\phi_{c_1},\phi_{c_m})\, p(\phi_{c_1},\phi_{c_n})}{p(\phi_{c_1})}\right] d\phi_{c_1}\, d\phi_{c_m}\, d\phi_{c_n} \tag{18}$$

where the pairwise joint pdf is given in [9] as

$$p(\phi_{c_1}, \phi_{c_i}) = \frac{\exp\left[\alpha_i \cdot \cos\left(\phi_{c_i} - \eta_{1i} \cdot \sqrt{\frac{\rho_{c_1}}{\rho_{c_i}}} \cdot \phi_{c_1}\right) + \rho_{c_1} \cdot \cos(\phi_{c_1})\right]}{(2\pi)^2 I_0(\alpha_i) I_0(\rho_{c_1})} \tag{19}$$

where η_{1i} is the correlation coefficient between the two carrier error processes and is given in [9] as

$$\eta_{1i} = \sqrt{\frac{\rho_{c_i}}{\rho_{c_1}}}\left[\frac{\xi_{1i}^2}{3} \cdot \frac{1 + 4\xi_{1i} + 3\xi_{1i}^2}{1 + 2(\xi_{1i} + \xi_{1i}^2 + \xi_{1i}^3) + \xi_{1i}^4}\right] \tag{20}$$

with $\alpha_i = \frac{\rho_{c_i}}{1-\eta_{1i}^2}$. Since by assumption, antenna 1 is the aiding antenna, then its carrier loop SNR, ρ_{c1}, is still given as before from (10). The loop SNR of all the aided antennas, 2 through L, is derived in [9], based on F-P techniques, and is shown to be

$$\rho_{c_i} = \left[\frac{1}{\rho'_{c_i}} + \frac{\xi_{1i}}{3\rho_{c_1}} \cdot \frac{2 + 4\xi_{1i} + 5\xi_{1i}^2 + 3\xi_{1i}^3}{1 + 2(\xi_{1i} + \xi_{1i}^2 + \xi_{1i}^3) + \xi_{1i}^4}\right]^{-1} \tag{21}$$

where the nominal carrier loop SNR is given by $\rho'_{c_i} = \frac{P_{ci}/N_{0i}}{B_{ci}}$, and $\xi_{1i} = \frac{B_{c1}}{B_{ci}}$ is the ratio between the aiding carrier loop bandwidth to that of the aided loop. Let us consider two extreme cases. First, as $\xi_{1i} \to 0$, i.e., when the bandwidth of the aiding antenna. is much smaller than the that of the aided antenna, then from (21), we find that $\rho_{c_i} \to \rho'_{c_1}$. That is, there is no advantage in aiding under this scenario, as expected. On the other hand, as $\xi_{1i} \to \infty$, which is the practical scenario, the effective loop SNR of the aided antenna is given from (21) as

$$\rho_{c_i} = \left[\frac{1}{\rho'_{c_i}} + \frac{1}{\rho_{c_1}}\right]^{-1} \tag{22}$$

In this case, as a benefit of aiding, the i^{th} loop is enabled to track which otherwise would not be possible. However, the tracking is accomplished with degraded performance. The effective loop SNR as seen in (22) is degraded from the nominal loop SNR by a factor of $\frac{\rho_{c_1}}{\rho_{c_1}+\rho_{c_i}}$. Ideally, when there are no phase errors (i.e., when $\rho_{c_i} = \rho_{sc} = \rho_{sy} = \infty$), $\overline{C_{c_i}^2} = \overline{C_{c_m,c_n}} = \overline{C_{sc}^2} = \overline{C_{sy}^2} = 1$ and the SSNR degradation as given in (14) becomes 0, as expected.

3.2 Symbol Error Rate Loss

The Carrier-Aiding/BBC SER for an L-element array, denoted $P_s(E)$, is defined as

$$P_s(E) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\phi_{c_1}}\int_{\phi_{c_2}}\cdots\int_{\phi_{c_L}} P_s'(E) \Big[p(\phi_{sc})p(\phi_{sy}) \times p(\phi_{c_1}, \phi_{c_2}, \cdots, \phi_{c_L})\Big] d\phi_{c_1} \mathbf{d}\boldsymbol{\phi}_{\mathbf{c_j}} d\phi_{sy} d\phi_{sc} \qquad (23)$$

where $\mathbf{d}\boldsymbol{\phi}_{\mathbf{c_j}} = d\phi_{c_2} d\phi_{c_3}\cdots d\phi_{c_L}$. The statistics of the synchronization phase error processes ϕ_{sc}, and ϕ_{sy} were described earlier. Whereas, the joint pdf of the carrier error processes can be simplified by first conditioning on ϕ_{c_1} and then by using Bayes' Theorem to the following form

$$\begin{aligned} p(\phi_{c_1}, \phi_{c_i}, \cdots, \phi_{c_L}) &= p(\phi_{c_1})\, p(\phi_{c_2}|\phi_{c_1})\, p(\phi_{c_3}|\phi_{c_1}) \cdots p(\phi_{c_L}|\phi_{c_1}) \\ &= p(\phi_{c_1}) \prod_{i=2}^{L} \left[\frac{p(\phi_{c_1}, \phi_{c_i})}{p(\phi_{c_1})}\right] \end{aligned} \qquad (24)$$

where the pdf of ϕ_{c_1} was given earlier in (8) and the pairwise joint pdf, $p(\phi_{c_1}, \phi_{c_i})$, was derived earlier in (19). The conditional SER is defined in [11] as

$$P_s'(E) = \frac{1}{4}\mathrm{erfc}\left(\sqrt{SSNR' \text{ when } d_k \neq d_{k-1}}\right) + \frac{1}{4}\mathrm{erfc}\left(\sqrt{SSNR' \text{ when } d_k = d_{k-1}}\right) \qquad (25)$$

where $\mathrm{erfc}(x)$ can expressed in terms of the error function, defined earlier, as $1 - \mathrm{erf}(x)$. For this scheme, the conditional SER is obtained by substituting the expression for the conditional SNR as given in (13) in the above expression, and making simple algebraic manipulations

$$P_s'(E) = \frac{1}{4}\mathrm{erfc}\left[\sqrt{(\sum_{i=1}^{L}\frac{E_{si}}{N_{0i}})C_{comb}C_{sc}(1-\frac{|\phi_{sy}|}{\pi})}\right] + \frac{1}{4}\mathrm{erfc}\left[\sqrt{(\sum_{i=1}^{L}\frac{E_{si}}{N_{0i}})C_{comb}C_{sc}}\right] \qquad (26)$$

where

$$C_{comb} = \frac{\sum_{i=1}^{L}\gamma_i C_{c_i}}{\sum_{i=1}^{L}\gamma_i} \qquad (27)$$

is the combined carrier degradation, and $\frac{E_{si}}{N_{0i}}$ is the symbol SNR received at antenna i. Again, as a check, when there are no timing errors (26) reduce to the well known binary phase shift keyed (BPSK) error rate of an array of L antennas, $P_s(E) = \frac{1}{2}\mathrm{erfc}(\sqrt{\sum_{i=1}^{L}\frac{E_{si}}{N_{0i}}})$.

4 Example & Discussion

Consider an array consisting of two different size antennas, 70-m antenna and 34-m antenna, operating in the S-band. The corresponding antennas' gamma factors are $\gamma_1 = 1$ and $\gamma_2 = 0.17$, respectively.

4.1 Tracking

The symbol SNR degradation corresponding to this scenario is determined from (14) and the theoretical as well as the simulated results arc shown in Figure 3. For example, when the reference antenna's SSNR value is at O dB, the total theoretical and simulated degradation values were -0.97 dB and -1.03 dB, respectively. The total degradation shown in Figure 3 include the degradation due to the subcarrier (SC) error process, the symbol (SY) timing error, and the carrier (CA) error processes. The conditional SER for two different size antennas, on the other hand, reduces to

$$P_s'(E) = \frac{1}{4}\left[\operatorname{erfc}\left[\sqrt{\left(\frac{E_{s1}}{N_{01}} + \frac{E_{s2}}{N_{02}}\right)C_{comb}C_{sc}\left(1 - \frac{|\phi_{sy}|}{\pi}\right)}\right] + \operatorname{erfc}\left[\sqrt{\left(\frac{E_{s1}}{N_{01}} + \frac{E_{s2}}{N_{02}}\right)C_{comb}C_{sc}}\right]\right] \quad (28)$$

where the combined carrier degradation , corresponding to this case, is given from (27) as:

$$C_{comb} = \frac{C_{c_1} + \gamma_2 C_{c_2}}{\gamma_1 + \gamma_2} \quad (29)$$

Substituting in (26) we get the conditional SER. Then the SER is determined by evaluating numerical] y the 4-tuples integral given in (23). The theoretical as well the simulation results arc depicted in Figure 4. From this figure, it is shown for example that when the reference antenna's SSNR value is O dB, the theoretical and simulated SER loss values are -1.05 and -1.09 dB, respectively. Table 1 summarizes the SER loss and SSNR degradation for several operating points.

4.2 Acquisition

This section considers a scenario where carrier aiding is required for signal acquisition and tracking. Consider the following scenario which is typical for the Galileo spacecraft mission to Jupiter. The carrier signal SNRS to be received at the above two co-located antennas are

~ = 14 .22 dB-Hz and $\frac{P_{C2}}{N_{02}}$ = 6.52 dB-Hz, respectively, and the carrier frequency predict error at both antennas is roughly 1 Hz. The loop bandwidth (BW) used to track the 1 Hz effective Doppler is at least 2112 [12]. Furthermore, since the minimum PLL loop SNR required for signal acquisition and tracking is about ? dB [1 O], it is clear that, on its own, the 34-m antenna would not be able to acquire the signal with a 2 Hz BW. However, with carrier aiding from the 70-m antenna, the 34-m can acquire the signal as shown in Figure 5. This figure was obtained by simulating the system shown in Figure 1 with I. = 2. In the simulation, B_{c_1} = 2 Hz, and B_{c_2} = 0.5 Hz. The initial PLL numerically controlled oscillator (NC()) frequency of the 70-m antenna was offset by 1 Hz from the input signal frequency, whereas the NCO frequency of the 34-m antenna was set to O Hz. The 34-m antenna I'LL frequency can be set to O Hz because the 70-m antenna reference signal can be used to completely track the frequency error at the 34-m antenna, which can also be expected to be 1 Hz due to our assumption of co-located antennas. Consequently, the 34-m antenna PLL is only required to track the phase difference between the 70 and 34-m antennas, which for the simulation is 1.22 rads.

5 Summary and Conclusion

This paper described and analyzed the performance of carrier aiding scheme employing cascaded PLLs which can be used to acquire deep space signals at very weak SNR. The joint pdf of the two cascaded phase error processes, which is essential for determining the system performance, was derived based on Fokker-Planck techniques. The tracking and acquisition performance were verified by simulation for an array of two unequal size antennas.

Acknowledgments

The authors would like to thank S. Million for his help with simulations

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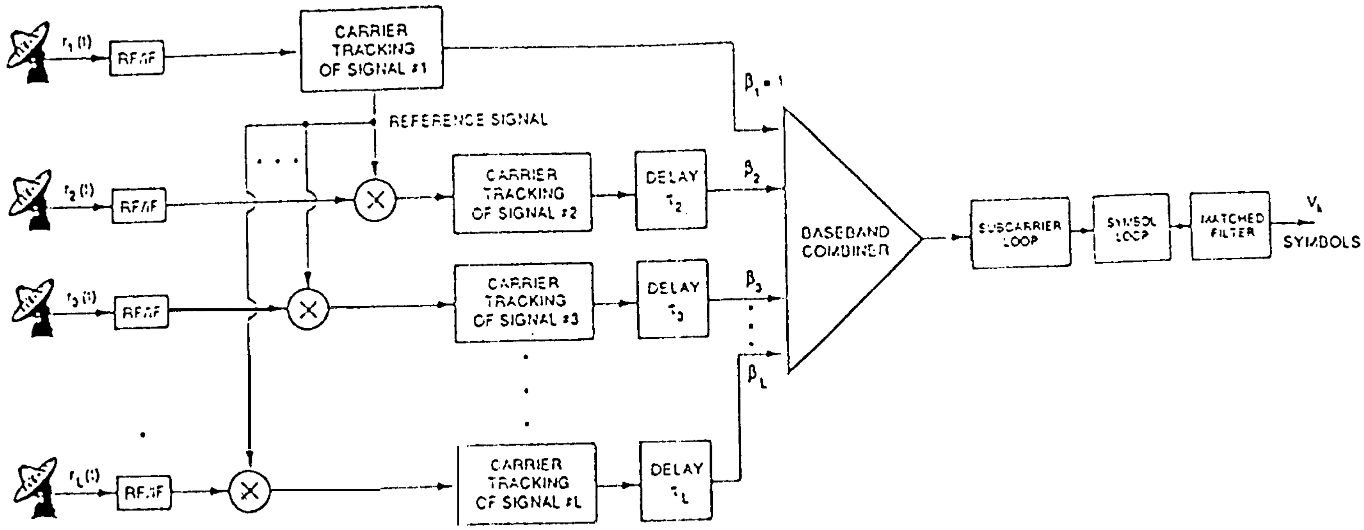

Figure 1: Carrier Aiding/BBC : System Overview

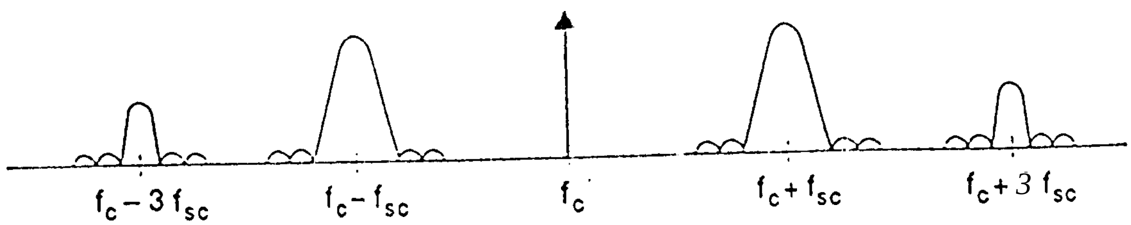

Figure 2: PCM/PSK/PM Square-Wave Subcarrier Signal Model

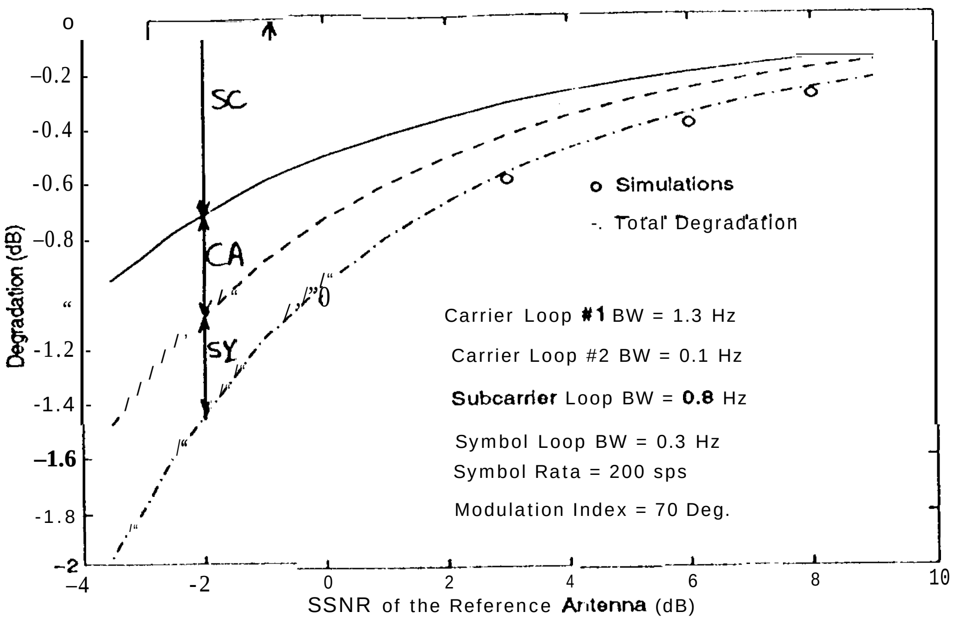

Figure 3: SSNR Degradation for Array of Two Different Antennas

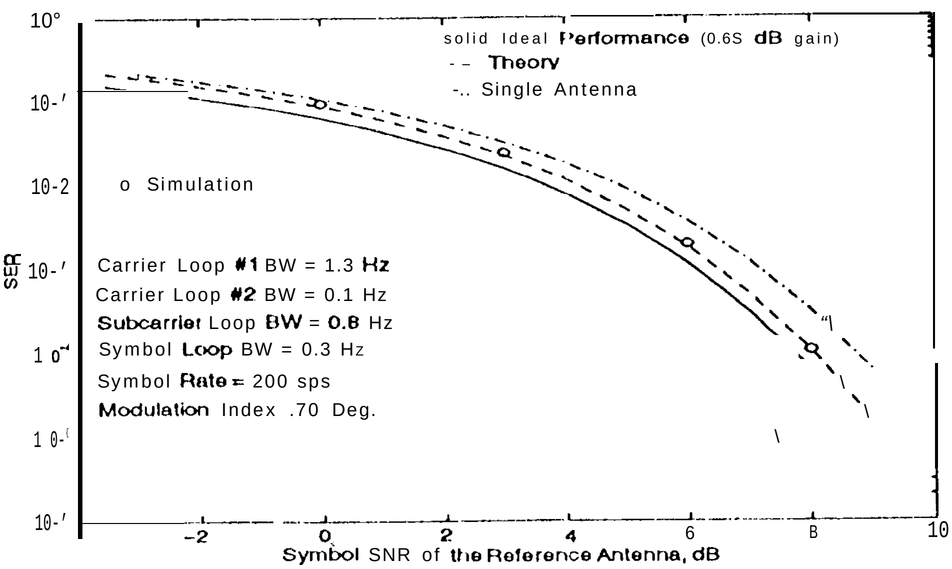

Figure 4: SER for Array of Two Different Antennas

Reference SNR (dB)	*SER Loss (dB)*		*SSNR Degradation (dB)*	
	Theory	Simulations	Theory	Simulations
0.0	-1.05	-1.09	-0.96	-1.03
3.0	-0.61	-0.63	-0.57	-0.6
6.0	-0.37	-0.36	-0.36	-0.4
8.0	-0.31	-0.29	-0.28	-0.3

'l'able 1: SER Loss vs. SSNR Degradation for an Array of Two Different antennas

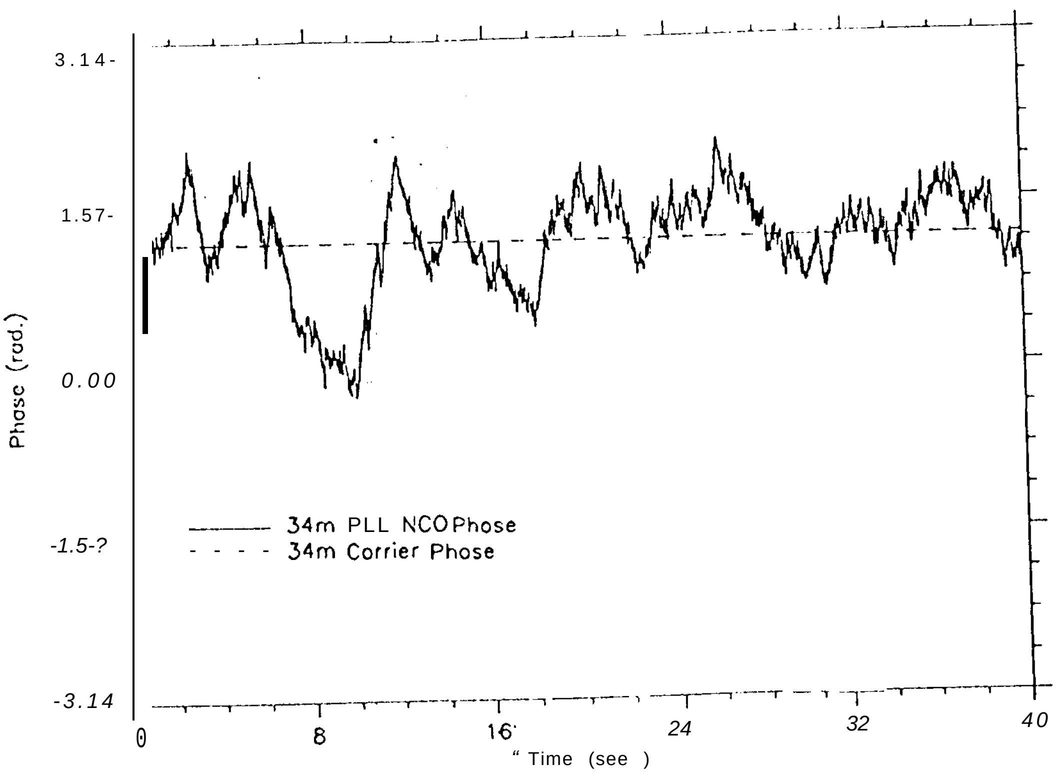

Figure 5: 34-m Antenna Carrier Phase Acquisition vs. Time